[*Transl*. This document has been translated from Chinese by David L B Jupp with valuable suggestions from Profs. Wang Chunmei of Northwest University in China and Li Lingtao of CSIRO in Australia. It was published in 1991 and is a valuable analysis of the projection used for the maps of China developed by Jesuit Brothers with the patronage of the Kangxi Emperor between 1704 and 1719. It has been translated to make its contents available to English readers and provide an accessible translated reference to this work. Some attention was given to the material with correction of small typographic and other errors and addition of some comments either in the text or as indicated by sentences enclosed in [*Transl*. square brackets]. The prefix "*Transl*." indicates interpolation by the Translator. Some work was done to clarify and enrich the references to European cartographers and there were some changes in structure at the beginning and end of the paper to conform more closely with the normal layout of western journals. In particular the "Statement of Purpose", "Acknowledgements" and "References" are changes to structure but only include original material. The references originally included as scattered footnotes have been collected in a References section at the back in English and Chinese and indicated in the text by numbered references of the form [R.n]. The Translator is a scientist and not a professional translator and the work will have consequent blemishes. However, it is believed to be an accurate presentation of the technical information in the original. Errors and confusing expressions are welcome to be brought to the attention of the translator who can be contacted HERE.]

A new investigation of the projection of the copper plate version of the Kangxi "Complete map of the imperial domain"

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Summary of Contents

The copper plate version of the Kangxi "Complete map of the imperial domain" has always been considered to be in a triangular projection (also called the trapezoidal projection). In this article, by using calculations and analysis based on data and measurements taken from the maps, it has been discovered that this map uses the "sinusoidal equal area pseudo-cylindrical projection" (also called the "Sanson Projection"). The article not only resolves the situation of the projection used, but puts questions such as whether the projection met the needs of mapping China, and the precision of locations of survey points on the map etc, on a sound footing. Furthermore, it has an important function in determining whether the survey methods for Latitude and Longitude were precise and allows analysis of areas in which the map is not precise.

Keywords: Complete map of the imperial domain, Sanson's projection.

Statement of purpose

The subject of this article is the set of copper plate maps discovered at the Shenyang Imperial Palace, called by Jin Liang “Complete map of the unified Qing Imperial territories of the Man and Han”, but in practice usually called “Complete map of the imperial domain”. The map is in 41 plates, each plate is 39.75cm long and 67cm wide. [R.1]. In the Kangxi period, modern survey and drafting methods were used to produce this map, and in the past it has generally been thought that the projection used was triangular [R.1] or trapezoidal [R.2] (see Figure 1), and although it has been disputed by some scholars, this question has still not had close examination or clear conclusion. [R.3]. Because this question is at the basis of the understanding and evaluation of the first mapping survey of the whole of China in Chinese history, it is necessary to study it deeply and aim to reach a clear conclusion. In this paper the conclusion to the projection problem posed is new.

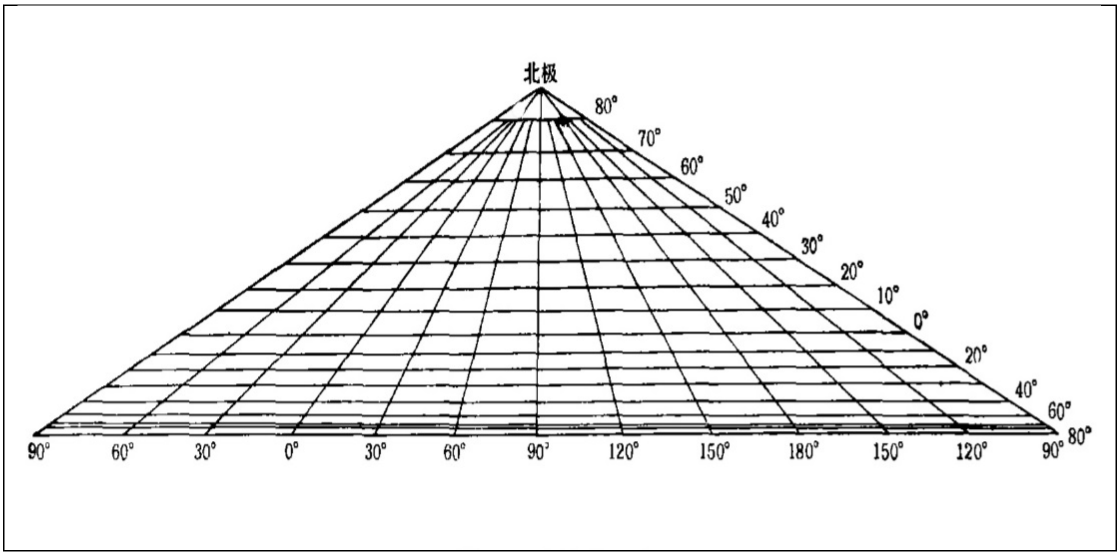

Figure 1 Diagram for the triangular (or trapezoidal) projection
(Beiji (北极) is the North Pole)

1. The characteristics of the projection used for the “Complete map of the imperial domain”

Properties of the projection

The convergence of the meridians

This paper makes use of 83 Latitude and Longitude intersections carefully selected from the “Complete map of the imperial domain” (see Table 1) to discover how, outside of the central meridian, from low Latitudes to high Latitudes, the angle of intersection between a given meridian and the parallels of Latitude progressively decreases. This is known as the convergence of the meridians. If the meridians are straight lines the angle of intersection is constant. If there were errors made at the time the plate was etched, it would also not be a regular progression. In Table 1, N is Latitude North, W is Longitude West and E is Longitude

East [*Transl*. of the Beijing meridian]. The theoretical values are as calculated from the formula for the Sanson Projection

Table 1 Theoretical and measured angles of Latitude and Longitude grid intersections

Lat	Lon	45W	40W	35W	30W	25W	20W	15W	10W	5W	0	5E	10E	15E	20E	25E	30E
	Theory						75.0	78.7	82.4	86.2	90.0	86.2	82.4	78.7	75.0	71.5	68.1
50N	Measured						75.0	79.0	82.5	87.0	90.0	86.0	82.0	79.0	75.5	73.0	67.0
	Diff						0	+0.3	+0.1	+0.8	0	-0.2	-0.4	+0.3	+0.5	+1.5	-1.1
	Theory	61.0	63.7	66.6	69.7	72.9	76.1	79.5	83.0	86.5	90.0	86.5	83.0	79.5	76.1		
45N	Measured	59.0	62.0	65.0	69.0	72.0	75.0	79.0	83.0	86.5	90.5	86.5	83.0	79.0	76.0		
	Diff	-2.0	-1.7	-1.6	-0.7	-0.9	-1.1	-0.5	0	0	+0.5	0	0	-0.5	-0.1		
	Theory		65.8	68.6	71.4	74.3	77.4	80.4	83.6	86.8	90.0	86.8	83.6	80.4	77.4		
40N	Measured		65.0	68.0	71.0	74.0	77.0	80.0	83.5	87.0	90.0	86.5	83.0	80.0	77.0		
	Diff		-0.8	-0.6	-0.4	-0.3	-0.4	-0.4	-0.1	+0.2	0	-0.3	-0.6	-0.4	-0.4		
	Theory		68.2	70.7	73.3	75.9	78.7	81.5	84.3	87.1	90.0	87.1	84.3	81.5	78.7		
35N	Measured		67.0	70.0	72.0	76.0	78.0	81.0	84.0	86.5	90.0	86.5	84.5	81.0	79.0		
	Diff		-1.2	-0.7	-1.3	+0.1	-0.7	-0.5	-0.3	-0.6	0	-0.6	+0.2	-0.5	+0.3		
	Theory	68.6	70.8	73.0	75.3	77.7	80.1	82.5	85.0	87.5	90.0	87.5	85.0	82.5			
30N	Measured	67.0	70.0	71.5	74.0	76.5	79.0	82.0	84.0	87.0	90.0	87.0	83.5	81.0			
	Diff	-1.6	-0.8	-1.5	-1.3	-1.2	-1.1	-0.5	-1.0	-0.5	0	-0.5	-1.5	-1.5			
	Theory		73.6	75.5	77.5	79.6	81.6	83.7	85.8	87.9	90.0	87.9-					
25N	Measured		73.0	75.0	77.0	78.5	82.0	83.0	85.0	87.6	90.0	88.5					
	Diff		-0.6	-0.5	-0.5	-1.1	+0.4	-0.7	-0.8	-0.3	0	+0.6					
	Theory					81.5	83.2	84.9	86.6	88.3	90.0	88.3					
20N	Measured					82.0	84.0	85.2	87.0	88.5	90.0	89.0					
	Diff					+0.5	+0.8	+0.3	+0.4	+0.2	0	+0.7					
	Theory									88.5	90.0						

[*Transl*. Table 1 is presented without a lot of explanation. The angles were measured from the map at the intersections of the parallels and meridians as indicated. The theoretical value is not discussed in any detail. It is the angle of the convergence of the meridians obtained using the definition of the Sinusoidal, or Sanson, projection model. The formula is provided by the author later as one of a list of properties of the projection but without reference to Table 1. Another note at that point will draw attention to it. At this stage the conclusion is simply that the longitudinal lines of the projection are curving lines that converge at the pole, are symmetric with regard to the standard meridian (Beijing) with systematically changing convergence of the meridians. It is enough for the author’s purposes.]

Spacing of parallels

The parallels of Latitude are not only straight lines, but also (close to) equal spacing for the 87 cases tested and analysed in these maps, the largest spacing found for these maps was 8.15cm and the smallest 7.82cm, and 30 have the value of 7.95cm, indicating Latitude spacing is likely to be a constant value (see Table 2), which will be taken to be 7.95cm. Because the differences from this “constant” value are distributed and do not follow a systematic pattern, whether a specific spacing is larger or smaller than this value is likely to be due to the engraving of the plate, deformations in the printing and errors of precision. By using a 100cm straightedge (greater than the width of any of the images) to check the latitudes, it was found they were straight lines. (In Table 2, Latitudes refer to the parallel between the spacing values under and to the left):

Table 2: The precision of the size of the spacing of Latitudes in the map

Lon	Latitude										
		20N	21N	22N	23N	24N	25N	26N	27N	28N	29N
40W							7.95	7.98	7.95	7.95	7.90
35W							7.95	7.98	7.90	7.96	7.90
30W							7.95	7.95	7.90	7.90	7.95
25W		7.85	7.86	7.90	7.88	7.95	7.88	7.90	8.00	7.90	7.94
20W		7.95	7.90	7.95	7.90	8.00	7.88	7.82	7.89	7.90	7.90
15W		7.90	7.92	8.00	7.95	8.05	7.94	7.95	7.94	7.95	8.00
10W		7.94	8.00	8.00	7.94	8.00	7.90	7.95	7.95	7.95	7.90
5W	7.95	7.92	8.00	8.00	7.95	8.15	7.95	7.95	8.00	7.95	7.92
0	7.95	7.95	7.92	7.95	7.92	8.00	7.99	7.95	7.98	7.95	7.98
5E		7.95	7.88	7.90	7.95	7.90	8.00	7.90	7.95	7.95	7.95

On the basis of the above characteristics, and after comparing them with the characteristics of a number of projections (see Table 3), it has been determined that the present projection belongs to the class of pseudo-cylindrical projections.

Table 3: Characteristics of the major types of map projection

Projection Name	Conic	Azimuthal	Cylindrical	Pseudo-Conic	Pseudo-azimuthal	Pseudo-cylindrical	Circular Conic
Forms of Latitude and Longitude							
Meridians	radial straight lines	radial straight lines	equal straight lines	symmetric curved lines	symmetric curved lines	symmetric curved lines	symmetric curved lines
Parallels	Parallel Concentric circles	Concentric circles	equal straight lines	circular arcs	concentric circles	straight lines	co-axial arcs

Specific characteristics	Meridians equal, Join in concentric Latitudes	symmetric shapes, far meridians compress equal Longitude difference	orthogonal Latitude and Longitude				

Projection distortions

Because the Latitudes and Longitudes of pseudo-cylindrical projections are not orthogonal, they cannot be an equal angle projection, so that it must be equal area or an arbitrary projection. Therefore, the projection of the "Complete map of the imperial domain" must be one of these. First of all we will investigate whether or not the map is equal area.

The conditions for an equal area projection are:

$$m \bullet n \bullet \sin\theta = 1$$

$$m \bullet n \bullet \cos\varepsilon = 1$$

$$\theta = 90^\circ - \varepsilon$$

In this formula:

m is the longitudinal proportional distance with value:

$$m = \frac{\Delta S'_m + \Delta S''_m}{2(\Delta S_m M_0)}$$

And ε is the complementary angle to the angle of intersection of Latitude and Longitude (θ);

n is the latitudinal proportional distance, with value:

$$n = \frac{\Delta S'_n + \Delta S''_n}{2(\Delta S_n M_0)}$$

and θ is the angle of intersection of Latitude and Longitude,

In these formulae: $\Delta S'_m, \Delta S''_m$ are the arc lengths of the latitudinal differences above and below the specific parallel of concern on the map that are averaged in *m*, $\Delta S'_n, \Delta S''_n$ are the arc lengths of the longitudinal differences to left and to right of the specific meridian of concern on the map that are averaged in *n*, $\Delta S_m, \Delta S_n$ are the differences on the earth between the same specific longitudinal and latitudinal lengths of a unit of arc difference, M_0 is the Map Scale.

$\varepsilon, \Delta S'_m, \Delta S''_m, \Delta S'_n, \Delta S''_n$ can be established directly by measurements from the map, but to establish $\Delta S_m, \Delta S_n, M_0$ you must make some further analysis and computations.

The values of $\Delta S_m, \Delta S_n$

To know these you must know the parameters of the sphere used in the survey. In a letter sent to the Church by the surveyor Brother Jean Baptiste Regis (Chinese name Lei Xiaosi, 雷孝思 Zi Yongwei, 永维, 1663-1736) he wrote:

“We finally decided to use a sphere with the meridian lines equally spaced, so that it would conform with the universally accepted view that the earth is a sphere....” [R.4]

As can be seen, at that time the figure selected for the earth was a sphere, with all radii equal. However, what radius was used for this sphere? The present writer has not found an answer to this in the literature, but it is possible to use other materials to infer it. We know: western science at the time had been based on a spheroid with 360 deg, and the knowledge that arose [in China] at the end of the Ming Period was on the same basis [R.5], thus if you know the length of each degree, you have an estimate of the circumference, and can therefore estimate the radius.

Fr. Regis also wrote:

“one degree is precisely 200 (Chinese) Li, every Li is 180 Zhang, every Zhang is 10 Chi, on the basis of measurements by the Paris Academy, each division of 1/20 of a degree is 2853 Toise, and 1 Toise is 6 Pied du Chatelet, and corresponds to 1800 Chinese Chi, or 10 Li. Computed on the basis of this scale, 1 degree corresponds to 20 of our Long Leagues (also called Marine Leagues), which is also the same as 200 Chinese Li.” [R.4]

Using established metric system equivalents for the Toise (1.949m), Pied du Chatelet (0.325m) and the Long League (5565m), we can make 3 estimates for the length of the Chinese Long Li:

$$2853 \times 20 \times 1.940 \div 200 = 556.0497(m)$$
$$2853 \times 20 \times 6 \times 0.325 \div 200 = 556.335(m)$$
$$20 \times 5565 \div 200 = 556.5(m)$$

Because the Long League is larger, the error will be smaller, so we will use the third value, or each Chinese Li being 556.5 metres, from which we can reach conclusions as shown in the following calculations:

Circumference of the Earth: 360 x 200 x 556.5 = 40068000
Radius of the Earth (R): 40068000 ÷ 2π = 6377020.3
Arc length of 1 degree of Longitude (ΔS_m): 200 x 556.5 = 111300
Arc length of 1 degree of Latitude at the equator (ΔS_{n_0}): 200 x 556.5 = 111300

ΔS_n can in principle be obtained using the formulae for spherical triangles.

[*Transl*. The radius of the Earth here may not be exactly that used by Fr. Regis which probably will stay unknown. It is the radius value from the definition of the Long League as the League of 20 to a degree arc, as measured and expressed in its value in Toise and in the standardisation to the metre in the late 18th century. But it is likely be quite close.]

The value of M_0 (map scale)

From Table 2 we know that that the distance between Latitudes is at most 7.95cm, which we will take as a constant, therefore the overall scale of the "Complete map of the imperial domain" (M_0) is:

Length on the map / actual length = 7.95/11130000 =1/1,400,000.

Equal Area

The total value of $m \cdot n \cdot \sin\theta$ has been measured and computed by the present author (see Table 4), using 8 successive situations for $m \cdot n \cdot \sin\theta$ and found to be close to 1. The smallest found was 0.99 and the largest was 1.03 [*Transl*. the median value is 1.0], proving that this projection is certainly an equal area projection. Furthermore, the value of n is close to 1 (smallest was 0.9974012 and the largest was 1.0015057), showing that the latitudinal distances have no length distortions. Then in Table 5 it can be seen that at the equator the meridians also maintain equal lengths ($m_0 = 1$).

[*Transl*. The Sinusoidal is, in fact, the only one that satisfies what is already known – being pseudo-cylindrical and equal area. But the proof is a little complex and the discussion by the author is very valuable regarding projections and their history. The proof of the above fact can be found in the added reference [R.9].]

From the above results it follows that the projection of this map is an equal area pseudo-cylindrical projection; but thus far, the problems are still not all resolved because we have not yet made clear what type of projection it is, what its mathematical formulae are or how to calculate it; and we have not made clear whether this projection was developed by the people who drew the map or if they used one that was already in existence.

Table 4 Table of ratios for the lengths of Latitudes and Longitudes and relative area distortion below 40deg Latitude

Lat	40N	35N	30N	25N	20N	15N	10N	5N
θ	65	68	71	74	77	80	83.5	87
$\sin\theta$	0.906308	0.927184	0.945519	0.961262	0.97437	0.984808	0.993572	0.99863
$\Delta S'_m$	8.7	8.6	8.3	8.2	8.2	8.1	8.1	8.0
$\Delta S''_m$	8.55	8.45	8.35	8.1	8.1	8.5	8.0	8.0
$\Delta S'_n$	6.1	6.1	6.05	6.1	6.1	6.1	6.1	6.1

$\Delta S''_n$	6.1	6.1	6.1	6.1	6.1	6.1	6.1	6.1
ΔS_m	11130000	11130000	11130000	11130000	11130000	11130000	11130000	11130000
ΔS_n	8527160	8527160	8527160	8527160	8527160	8527160	8527160	8527160
m	1.0849059	1.072327	1.0471698	1.0314465	1.0251572	1.0440251	1.0125785	1.0062893
n	1.0015057	1.0015057	1.0015057	1.0015057	1.0015057	1.0015057	1.0015057	1.0015057
P	0.99	1.00	0.99	0.99	1.00	1.03	1.01	1.01

Table 5 Table of ratios of meridians to Central meridian for sphere

φ	20N	25N	30N	35N	40N	45N	50N
θ	90	90	90	90	90	90	90
$\sin\theta$	1	1	1	1	1	1	1
$\Delta S'_m$	7.95	7.90	7.90	7.90	8.00	7.95	7.95
$\Delta S''_m$	7.92	7.95	7.95	7.90	7.90	7.95	7.95
ΔS_m	11130000	11130000	11130000	11130000	11130000	11130000	11130000
m_0	1.00	1.00	1.00	0.99	1.00	1.00	1.00

2. The background and development of western studies in map projection

In working to resolve the above problem, it is valuable to investigate the background and development of projections in western cartography. The birth of western science of map projections was in Ancient Greece. Dicaearchus (350-290 BC, 迪亚西库) in his book "Description of the Earth" (end of the 4th Century BC) had already used the square grid representation. Eratosthenes (276-194 BC, 埃拉托色尼) in his book "A treatise on Geography" also described the theory of drawing maps of the inhabited world. In Ancient Roman times, the famous scholar of classics Ptolemy (Second Century AD, 托勒玫) in his book "Geographica" expounded the theory of map projections and included maps, he used several kinds of projection, among them having two important kinds: one is the circular conic projection in which the Longitudes are straight lines radiating out from the vertex (north) and Latitudes are circular arcs, in addition, near the vertex, they are like concentric circles around it; the other is the amended circular conic projection in which, except for the central meridian which is a straight line, the meridians form symmetric curving lines.

In the 16th century, cartographers started to use projections on a spherical surface, this type of projection can be divided into three categories as "polar (conic) spherical projections", "equatorial (cylindrical) spherical projections" and "oblique (azimuthal) spherical projections". In 1507, Walter Lud first employed the "polar spherical projection", in 1540 Gemma Frisius (1508-1555, 弗里修斯) first used the equatorial spherical projection and in

1514 Johannes Stabius (Stab) (斯太伯) and Johannes Werner (1466-1528, 威纳) first used the oblique spherical projection. In the 16th and 17th centuries, the most famous of the projections was the Mercator Projection. This was first used [empirically] by the Dutchman Gerhard Mercator (1512-1594, 墨卡托) in 1569 to draw marine charts. The projection was later computed (into tables) by the Englishman Edward Wright (1558－1615, 赖德) in 1599 and 1610 with the mathematical formulae for the projection being established in about 1668 by James Gregory (1638-1675, 格利哥利). It was one type of vertical cylindrical projection, with Latitudes and Longitudes being orthogonal sets of straight lines.

Another type of projection (developed) was the Bonne Projection, which was previously suggested by a map in Ptolemy. Mercator (also) used it earlier in 1595 for maps of Africa and Asia, which led Germans to call it the “Mercator equal area projection”. Rigobert Bonne (1727-1795, 彭纳) resolved how to calculate this projection using the mathematics of the angles involved, leading to it also being called the “Bonne Projection”. This name has continued to be used since. The Bonne projection is of the pseudo-conical equal area projection type, it has 4 special properties: (1) the central meridian is an un-deviating straight line; (2) the Latitudes are equi-distant parallel concentric circles; (3) the central meridian and every Latitude, and the central Latitude and every meridian, are orthogonal; (4) it is equal area. In 1606 Mercator also used what was later called the “Sanson Projection” to draw his map of South America. Nicolas Sanson d’Abbeville (1600-1677, 桑逊) used it in 1650 to draw all kinds of maps, leading to it being (often) called the “Mercator-Sanson Projection”, in 1729 the Englishman John Flamsteed (1646-1719, 弗兰斯蒂) also used it to draw maps, causing to be further referred to as the “Sanson-Flamsteed Projection”, but it has been more often (simply) been called the “Sanson Projection”. It is a pseudo-cylindrical equal area type of projection, its most important properties are: its meridians are represented as sine curves, the central meridian and equatorial (radius) are equal and orthogonal straight lines, parallels of Latitude are equally spaced parallel lines, it is equal area.

In 1613 Francois d'Aguilon (1567-1617, 阿奎伦) made use of a radiating line (stereographic) projection to draw maps, this type of projection had mainly been used to draw celestial maps, lunar phases etc. In 1745 the Frenchman Cesar Francois Cassini (1714-1784, 卡西尼) used the “Cassini Projection” to draw the 1:86400 scale field survey Atlas of France, this projection is most useful for large area surveys but has not been used widely in map making. The following years saw the development of the “Lambert Projection” in 1772, the “Mollweide Projection” in 1805 and the “ordinary polyconic projection” in 1820 [R.6]. From the above can be seen that among map projections developed before the “Complete map of the imperial domain” was drawn, ie before 1718, the characteristics of the “Sanson Projection” used in 1650 by Sanson and those established for the “Complete map of the imperial domain” (seem to) conform the best.

3. Representation and calculations for the “Sanson Map Projection”

How can we establish whether the “Complete map of the imperial domain” is in the Sanson Projection or not? To make the answer in the affirmative, we must show that using the formulae of the projection to compute various characterising features and using equivalent measurements on the “Complete map of the imperial domain” come to the same result.

Formula and calculations for the Sanson Map Projection [R.7]

Projection Formulae:

$$X = R\varphi$$
$$Y = R\lambda\cos\varphi$$

Parameter Values:

$$n = 1$$
$$m = \sec\varepsilon$$
$$P = 1$$
$$\mathrm{Tan}\,\varepsilon = \lambda\sin\varphi$$
$$\mathrm{Tan}\frac{\omega}{2} = \frac{1}{2}\lambda\sin\varphi$$

In these equations, X is the projection ordinate, Y is the projection abscissa, φ is map Latitude, λ is map Longitude (with zero being the standard meridian, which in this case is Beijing), R is the Earth Radius, *n* is the scale in the Latitudinal direction, *m* is the scale in the Longitudinal direction, P is the relative surface area, ε is the complement of the angle of intersection between Latitude and Longitude, ω is the greatest angular distortion. The values of three of these have already led to conclusions for the "Complete map of the imperial domain": $n = 1$, $P = 1$ & $m_0 = 1$. That is, these three measurements conform with the equations. If we can prove that the X and Y values also conform, it follows that the projection used by the "Complete map of the imperial domain" is the Sanson Projection without any doubts.

[*Transl*. The equation in the above list $\mathrm{Tan}\,\varepsilon = \lambda\sin\varphi$ provides the theoretical value for the angle of the convergence of the meridians computed by direct measurement in Table 1. There is no explanation of the formula here but it is possible there was in the Thesis from which the paper has been extracted. Table 1 therefore also provides additional evidence for the projection having the above form.]

Value of X

Following on from the above, *R* is 6377020.3 metres, M_0 is 1/1,400,000, therefore:

$$X = R\varphi = 6377020.3 \times \varphi \times 1/1.4M = 7.95 \times \varphi$$

Therefore the form is known, and according the map projection, the distances between parallels must be equal and equal to 7.95 cm. From Table 2 it can be seen that the "Complete map of the imperial domain" has identical Latitude spacing with average value 7.95 cm, indicating $X = R\varphi$ conforms with this map.

Value of Y

Following on from the above, we have:

$$Y = R \times \cos\varphi \times \lambda = 7.95 \times \lambda \times \cos\varphi$$

Considering Table 6, where the calculations are all made at a Latitude of 40 degrees North, it can be seen that if we round to one decimal place, then (to that accuracy) the hypothesis that ΔY is equal to $\Delta Y'$ is confirmed, so it is also confirmed that $Y = R \times \cos\varphi \times \lambda$ conforms with the practical situation of the "Complete map of the imperial domain". In the Table, λ is Longitude, Y is the value of the abscissa, ΔY is the theoretical Longitudinal distance, $\Delta Y'$ is the longitudinal distance on the map, units are in cm.

Table 6: Comparative Table of distances between Longitudes at Latitude 40deg

λ	4	5	6	9	10	11	14	15	16	19	20	21
Y	24.36	30.45	36.54	54.81	60.9	67	85.26	91.35	97.44	115.71	121.8	127.88
ΔY		6.09	6.09		6.09	6.10		6.09	6.09		6.09	6.08
$\Delta Y'$		6.10	6.10		6 10	6.05		6.10	6.10		6.10	6.10
λ	24	25	26	29	30	31	34	35	36	39	40	41
Y	146.16	152.25	158.34	176.61	182.7	188.79	207.06	213.15	219.24	237.51	243.6	249.69
ΔY		6.09	6.09		6.09	6.09		6.09	6.09		6.09	6.09
$\Delta Y'$		6.1	6.1		6.05	6.1		6.1	6.1		6.1	6.1

The outcomes discussed above completely prove that the projection used for the "Complete map of the imperial domain" is the "Sanson Projection" (also called the "sinusoidal pseudo-cylindrical equal area projection") (see Figure 2)

Since in the Sansong Projection in the vicinity of the central Latitudes and Longitudes there is little distortion, it is therefore quite suitable for regions which are equatorial or where the central meridian is not very far from others (such as South America, Africa or Australia etc). It seems (therefore) that if it is used in mid-Latitudes, the east and west Longitudes should not be far apart, but if it is used in high Latitudes the distortions will be large. On the basis of projection theory, it would seem that in the area where the "Complete map of the imperial domain" is drawn, the most suitable projections would be of conic type such as Lambert Projection, Albers Projection etc. However, in 1718 these two projections had not yet come into being. Comparatively speaking therefore, among the available projections, selecting the Sanson Projection to draw a map of China is probably quite suitable.

4. Use of the Map Projection formulae to analyse Latitudes and Longitudes of survey points in the map

Since it seems that no records of Latitudes and Longitudes from the large scale survey of the Kangxi Period remain in Chinese Language, scholars believe that the Latitudes and Longitudes of 641 places provided in the Frenchman J.B. Du Halde's [F.8] book were those meant to be used with the “Complete map of the imperial domain”. [However,] according to the calculations by the present author not all of the points in the book were used and not all of the coordinates used for the maps were those in the books.

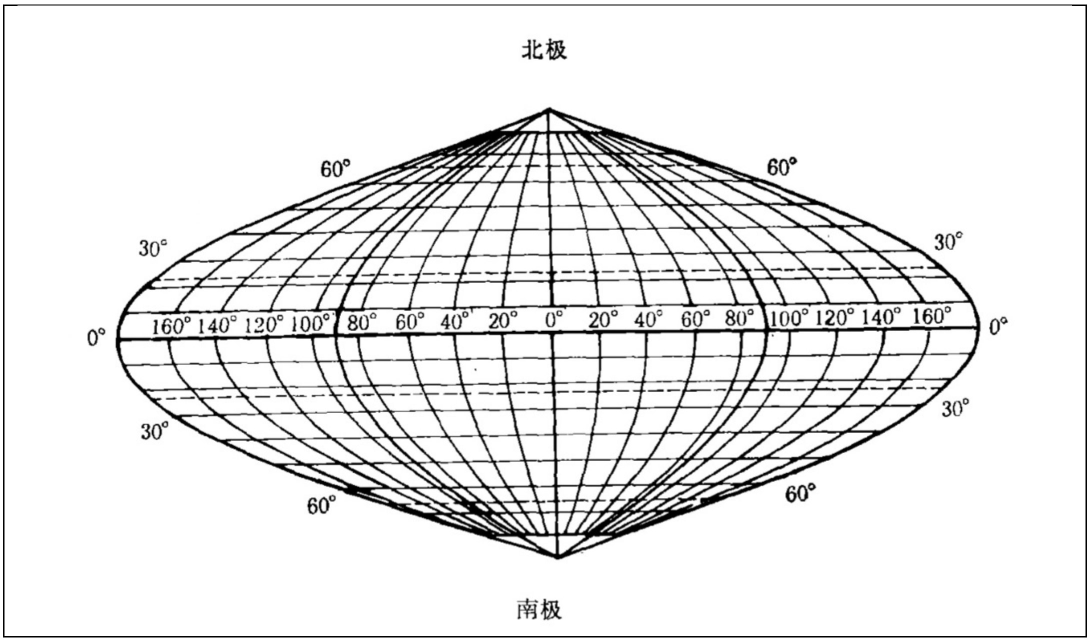

Figure 2: Diagram for the Sinusoidal equal area pseudo-cylindrical projection (Beiji (北极) is the North Pole and Nanji (南极) is the South Pole)

In Table 7, Latitude and Longitude values are restricted to the 15 Provinces within the Great Wall within which there are 493 places of the total of 641.

Table 7. List of differences between the coordinates of Latitudes and Longitudes from du Halde of 15 Provinces and map locations

Province	# Points	Found	Longitudinal Differences			Latitudinal Differences			Longitude abs error distribution			Latitude abs error distribution		
			Zero	Positive	Negative	Zero	Positive	Negative	≤ 0.2cm	0.2 ≤,≤ 0.5cm	>0.5 cm	≤ 0.2cm	0.2 ≤,≤ 0.5cm	>0.5cm
直隸	48	45	12	17	16	17	9	19	37	6	2	36	3	6
江南	38	37	15	16	6	5	10	22	33	2	2	36	0	1
江西	30	30	11	14	5	12	7	11	29	1	0	28	2	0
福建	37	36	10	18	8	11	5	20	30	5	1	32	3	1
浙江	30	30	8	16	6	14	9	7	27	1	2	28	1	1

湖廣	54	54	12	33	9	15	7	32	38	13	3	47	5	2
河南	29	29	7	17	5	11	9	9	20	7	2	27	0	2
山東	28	28	9	8	11	8	6	14	26	1	1	23	1	4
山西	28	27	3	22	2	7	13	7	20	6	1	25	0	2
陜西	28	28	0	27	1	6	12	10	6	19	3	25	2	1
四川	28	28	1	27	0	3	3	22	3	19	6	26	2	0
廣東	45	42	3	36	3	4	3	35	21	17	4	36	5	1
廣西	28	25	1	23	1	3	2	20	15	7	3	21	2	2
云南	30	30	1	28	1	7	4	19	7	17	6	21	6	3
貴州	25	24	1	21	2	5	1	18	7	13	4	20	2	2
Total	506	493	94	323	76	128	100	265	319	134	40	431	34	28
%	100	97.0	19.1	65.5	15.4	26	20.3	53.4	64.7	27.2	8.1	87.4	6.9	5.7

From Table 7 can be established:

1. Among the 493 points where Latitude and Longitude coordinates are located, the Longitude coordinates which are (near) zero are 94 or 19.1%, coordinates that are positive are 323 or 65.5% and those that are negative are 76 or 15.4%.

2. Among the 493 points those where differences in Latitude were (near) zero are 128 or 26%, those that were positive are 100 or 20.6% and those that were negative are 265 or 53.4%. Because there are more (near) zero and negative values for Latitudes than for Longitudes then of course the number of positive differences for Latitudes is less than that for Longitudes. However, points 1 and 2 together suggest that the Latitude coordinate accuracy is higher than that of the Longitude. [*Transl*. Possibly with some bias in the case of Latitudes].

3. The Latitude and Longitude coordinates that differ by less than 0.2 cm absolute value are most likely from the du Halde book with plotting errors due to copper plate distortions or paper stretch. Those greater than 0.2 cm and less than 0.5 cm are possibly also from the book with errors due to these factors, or perhaps were different coordinates from those in the book. When they are greater than 0.5 cm it is more likely that the coordinates listed in the du Halde Table are not the ones plotted in the maps.

4. The absolute differences of Longitude coordinates that are less than 0.2 cm number 319 or 64.7% and for Latitude the equivalent is 431 or 87.4%, which suggests that Latitudes are relatively more accurate than Longitude. Those for which the absolute difference is greater than 0.2 cm and less than 0.5 cm Longitude number 134 or 27.2% and for Latitude the equivalent is 34 or 6.9%. Most of the errors in this region must be plotting errors which further suggests that Latitudes are relatively more accurate than Longitudes.

5. Absolute values greater than 0.5cm correspond to differences in Latitude and Longitude of more than 3.8 sec. Such large errors are unlikely to be plotting errors, especially if the difference is above 1 cm. For example, Gaogezhuang had a difference of 7.9 cm, and Shacheng's Longitude coordinate difference was 5cm. It would seem that making such an error [randomly] is improbable. The number of errors greater than 0.5cm reached 40 or 8.1% among Longitudes, and 28 or 5.7% among Latitudes with occasional values often being much

more than this. It is hard to see how the surveyors could have been mixed up to such a degree, so it is reasonable to suggest that some other coordinates have been used rather than those listed in the Tables.

[*Transl*. It is possible that the Copperplate map was not the primary map but rather derived from more basic maps where the points from the Du Halde Tables were all accurately plotted. This would explain the inaccuracies and missing points and should be a subject for further research.]

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ENGLISH ABSTRACT (from original paper with a few grammatical changes)

A NEW SCRUTINY OF THE TYPE OF PROJECTION IN HUANG YU QUAN LAN TU

Wang Qianjin

Abstract

It has previously been generally held that the projection of HUANG YU QUAN LAN TU (Complete Atlas of the Imperial Domain) printed from copperplates during Kangxi's reign is trapezoidal. Through detailed calculation and analysis, this paper holds that the Sanson projection is used for the Huang Yu Quan Lan Tu, and solves such issues as precision and fitness of the projection, precision of plotting of survey points, precision of map compilation and origin of geographical Longitude and Latitude adopted in the map.

Key words Huang Yu Quan Lan Tu, Sanson projection

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